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Nurturing Mathematical Creativity in Schools

Okullarda Matematiksel Yaratıcılığın Geliştirilmesi

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Abstract

Purpose of the article is to find possible answers on how to best nurture mathematical creativity of students. With the worldwide introduction of mandatory national/state tests and inclusion of diverse students in classrooms, most mathematics teachers lead the mathematical lesson by giving direct instruction on the procedures for math problem solving. Mathematics instructions are focused on getting good scores on tests of performance. To get insights on how to improve our educational practices to nurture mathematical creativity of students, conceptual models and multifaceted nature of mathematical creativity and its development are reviewed. Keeping in mind the need of a balanced development of various components of mathematical creativity, it is necessary to allow problem choices through differentiation of challenge levels of problems and contents, creating safe environment for taking risks, providing opportunities for recognition of mathematical creativity, and providing ill-defined challenging real life problems are suggested.

Keywords: mathematical creativity; multifaceted, mathematics education, school-level, professional level

Öz

Bu çalışmanın amacı öğrencilerin matematiksel yaratıcılık düzeylerinin nasıl geliştirileceğine yönelik olası yollar önermektir. Dünya genelinde ulusal testlerin zorunlu olması ve sınıflarda farklı düzeylerde öğrencilerin bulunması nedeniyle, çoğu matematik öğretmeni matematiksel problemlerin çözümünde doğrudan öğretim yöntemlerini kullanmaktadırlar. Matematik öğretiminin odak noktası sınavlarda yüksek puan almasıdır. Öğrencilerin matematiksel yaratıcılık düzeylerinin geliştirilmesine yönelik eğitim uygulamalarının tasarlanması kapsamında; kavramsal modeller ile matematiksel yaratıcılığın çok yönlü doğası ve gelişimi incelenmiştir. Matematiksel yaratıcılığın farklı bileşenlerinin dengeli gelişimi göz önünde bulundurularak, problem seçeneği oluşturma yolları olarak; problemlerin ve içeriklerinin zorluk düzeylerinin değiştirilmesi, risk alabilmek için güvenli ortamların yaratılması, matematiksel yaratıcılığın ortaya çıkması için fırsatlar yaratılması ve tam tanımlanmamış gerçek hayat problemlerinin tasarlanması önerilmiştir.

Anahtar Sözcükler: matematiksel yaratıcılık, çok yönlülük, matematik eğitimi, okul seviyesi, profesyonel seviye

"Mathematics is the science demanding the utmost imagination"
Sofya Kovaleskaya

Introduction

Mathematics is the foundation of science and technology (US. National Research Council [US. NRC], 1989) and the main driver of economic development (Chung, 2015). Thus, highly

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creative talents in mathematics have been proclaimed as the most needed resource for the 21st century as a highly creative workforce in Science, Technology, Engineering, and Mathematics (STEM) areas (US Office of Science and Technology Policy [US OSTP], 2006).

For the last one or two decades, Singapore (Gob, 1997), South Korea (Cho, 1995), and Taiwan (Chen, 2004) have made the fostering of creativity in the schools a top priority. In the United States, the Principles and Standards of School Mathematics (National Council of Teachers of Mathematics [NCTM], 2000) suggests that students need to be provided with challenging problems that can stimulate their development of creative mathematical thinking (Robelen, 2012). Nevertheless, our education system has not sufficiently responded to promote children's mathematical creative thinking during training process at school (Chan, 2007; Mann, 2005).

In this article, theoretical framework of mathematical creativity and mathematical creativity in professional mathematicians, multi-faceted nature of creativity, and development of creativity are reviewed. Based on the review of literature, strategies for nurturing mathematical creativity in schools are suggested.

Definition of Mathematical Creativity

In this section, multi-faceted nature of mathematical creativity is reviewed to answer such questions as 'What is mathematical creativity?'; 'What is involved in mathematical creativity?'; and 'How can we nurture mathematical creativity?'

Mathematical creativity may be present at different levels in all people (Liljedahl & Sriraman, 2006; Usiskin, 2000). Hence, until now, there have been not less than 100 definitions of creativity and mathematical creativity from various viewpoints (Treffinger, Young, Shelby,& Shepardson, 2002). However, not many studies have reached insights into the characteristics of mathematical creativity and how we can promote mathematical creative talents.

Common concept of creativity among various definitions are new, high quality, and appropriate ideas, products or performances within the context (Amabile, 1996; Sternberg & Lubart, 1995; Torrance, 1972, 1987). This definition of creativity can be extrapolated to the domain of mathematics as to produce new, elegant, and appropriate solutions for the problem (Ervynck, 1991), even though the specific descriptions of mathematical creativity may be slightly different among scholars.

For professional mathematicians, intuition on deep structure of the subject at a high degree helps an individual explore the hidden harmonies and relations of mathematics (Ervynck, 1991; Poincaré, 2012). However, intuition is only possible when the individual has acquired solid knowledge and skills, become familiar with the particular subject or problem and reflected deeply on it (Liljedah l& Sriraman, 2006). Furthermore, Poincaré (2012) described mathematical creativity as a work of both conscious and unconscious thinking in which consciousness comes first and unconsciousness appears later. Likewise, Kovaleskaya (1978) compared the mathematician to a poet, because like a poet, a mathematician must be creative to achieve mathematics magic. She found that the mathematician must identify what others cannot and think more intensely than others.

Chamberlin and Moon (2005) considered divergent thinking as one of the keys for mathematical creativity. Mingus and Grassl (1999) also suggested the ability of combining the experience and skills from remote domains to synthesize new products or ideas as mathematically creative ability. Poincaré (2012) added the ability to construct new and valuable combinations of mathematical entities that already exist as another essential ingredient of mathematical creativity. To create such combinations, one needs to be capable of associating remote concepts and skills in mathematics. Liljedahl and Sriraman (2006) echoed Poincaré that, while solving problems, one may have random pieces of ideas from which one searches for relevant ideas and merges them together meaningfully to create mathematics.

Livne and Milgram (2006), in their experimental study with students, found that mathematical creativity requires two cognitive abilities: Academic and creative abilities. Academic ability is considered to be mathematical thinking ability, while creative ability is the ability to recognize pattern and relationships using complex and non-algorithmic thinking and to think divergently. The results imply that students need to be equipped not only with creative abilities, but also with solid foundation of mathematical concepts and skills in order for them to be creative in mathematical problem solving.

From the review of mathematical creativity at the professional level, it is possible to sense that mathematical creativity requires various components including intuition, conscious and unconscious thinking, association of remote ideas, divergent thinking. Below, multi-faceted nature of mathematical creativity will be reviewed.

Multi-faceted Nature of Mathematical Creativity

Regarding the multi-faceted nature of creativity, Amabile's (1983a; 1983b) componential theory informs about three components involved in the framework of creative performance: *domain-relevant skills*, *creativity-relevant skills,* and *task motivation*. *Domain-relevant skills* include knowledge, skills, and "talent" of specific domain in which the problem to be solved belong. *Creativity-relevant skills* refer to knowledge and skills needed for generating novel ideas and working styles. *Task motivation* refers to the intrinsic and extrinsic motivation towards the task. Furthermore, motivation may be the most salient factor of creative performance (Amabile, 1985).

Sternberg and Lubart (1995) also proposed five personal resources of creativity: intelligence, knowledge, thinking styles, personality, and motivation. Knowledge refers to domain knowledge and formal and informal knowledge required for creating new and useful solutions. In addition, thinking styles are related to the theory of mental government. Sternberg claimed that creativity style is related to a person's self-government style where mental government can be categorized as functions of mental government (legislative, executive, and judicial styles), forms of mental self-government (monarchic, hierarchic, oligarchic, and anarchic styles), scope of mental self-government (internal and external styles), and orientations of mental self-government (liberal style, conservative style). Similar to other theories, creativity is also connected to both intrinsic and extrinsic motivation.

Urban's (2003) componential model of creativity that contains six components which are involved in the creative process: (1) divergent thinking and acting, (2) general knowledge and thinking base, (3) specific knowledge base and specific skills, (4) focusing and task commitment, (5) motivation and motives, and (6) openness and tolerance of ambiguity. According to Urban, no single component can work independently in the creative process; instead, all function interactively. In other words, each component and sub-component interact differently depending on the kind of problems, stage of creative processes, kind of processes in relation to the product strived for. Moreover, micro- and macro-environmental variables, such as the context of work, task constraints, evaluation, education, competition, cooperation, home climate, school climate, organizational climate, and social ambiance, also influence creativity.

Theoretical frame of creativity with emphasis on creative problem solving was suggested by Cho (2003) in her Dynamic System Model of Creative Problem Solving. It includes six attributes (divergent thinking, convergent thinking, motivation, general knowledge and skills, domain specific knowledge and skills, and environment) that interact with and are influenced by one another. Cho (2003) believed that, when students solve problems, motivation, knowledge and skills in general and in specific domains function as the base of creativity, while divergent and convergent thinking function as tools for creativity. Similar to Hocevar (1979), Cho (2003) found that ideational fluency is correlated highly with ideational originality. In other words, those who can come up with more diverse solutions tend to produce more original ideas. Based on the Cho's Dynamic System Model of Creative Problem Solving, Lin (2018) found "Threshold effects" with 409 5^{th} and 6th Taiwanese students in mathematics creative problem solving. A Creative Problem-Solving Attribute Instrument (Lin & Cho, 2011) was used to measure students' perceptions on their motivation, knowledge, and skills, both in general and in specific domains, divergent and convergent thinking. Cluster analyses yielded three creative problem-solving typologies: High, Medium, and Low. The High Attribute group scored significantly higher in the Math Creative Problem-Solving Test than did the Medium and Low Attribute groups. Interestingly, Medium attribute group was not significantly better than the Low attribute group in math creative problem solving ability. This result shows a threshold effect from the related attributes—divergent thinking, convergent thinking, motivation, general knowledge and skills, domain-specific knowledge and skills, and environment—on students' creative problem solving abilities. The result also implies that the balanced development of all related attributes is important for exercising creativity in children. The result also confirms the claim that none of the creativity components is solely responsible for mathematical creativity (Amabile, 1996; Cho, 2007; Sternberg & Lubart, 1995; Urban, 2003).

Development of Mathematical Creativity

Mathematical creativity can be discussed at four different levels based on the 4C Model of Creativity (Kaufmann & Beghetto, 2009). In 4C Model, 'Big-C' creativity refers to ideas bring about significant change in a domain (e.g., a mathematical discovery made by Fields medal recipients); 'Pro-c' creativity refers to the ideas associated with the creative acts of people with expertise in a field (e.g., professional mathematicians' ideas published in scholarly journals); 'little-c' creativity refers to the everyday creative acts of individuals who are not particularly expert in a situation (e.g., mathematical rediscovery of secondary school students in AP mathematics); and 'mini-c' creativity refers to the novel and personally meaningful interpretation of experiences actions and events made by individuals (e.g., young child's discovery of mathematical patterns from Lego blocks).

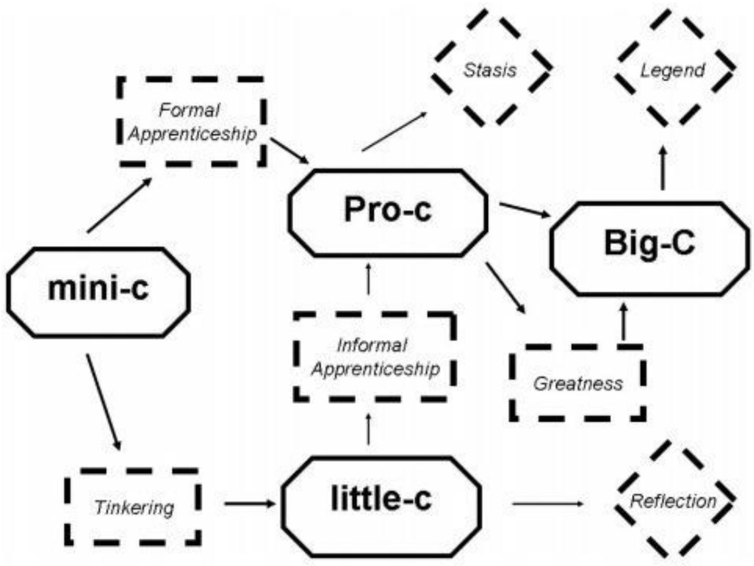

Figure 1. The Complete Four-C Model

The Complete Four-C Model in Figure 1 shows that "everyone begins in mini-c". Even if there is a "rare few may make the jump to Pro-c", some will reach to Proc-c through "a *formal apprenticeship*", usually taking approximately 10 years through academic institutions. Some will develop their creativity through 'tinkering—playing with one's creativity in a domain and improving through such experimentation, even without a structured mentorship' (Kaufmann & Beghetto, 2009, p. 7).

Depending on the potential and learning opportunities, each individual may demonstrate or follow different developmental trajectories. As can be seen in the Complete Four-C Model, some students may end up at the lttle-c level with reflection, rather than pursuing further up to the Pro-c/Big-C levels. Likewise, Liljedahl and Sriraman (2006) also distinguished mathematical creativity at the professional level and school level: Big-C/Pro-c at professional level and mini-c/little-c at school level of mathematical creativity. They conclude that Big-C/Pro-c mathematical creativity of professional mathematicians can be defined as the ability to create a unique work which remarkably advances mathematics or to suggest new questions and solutions for other mathematicians. Nevertheless, mini-c/little-c mathematical creativity of students is to solve a familiar problem from new perspective or provide an original solution to an assigned problem.

Hadamard (1954) claimed that the difference between mathematical creativity of Pro-c of mathematicians and little-c of students is only the degree or quality. Krutetskii (1976) agreed that creativity is not a patent of the mathematician. He claimed that mathematical creativity requires the ability to synthesize mathematical material; to find the simplest way to solve problems; to remember relation patterns, reasoning schemas, and methods of problem solving. In addition, being flexible in reasoning processes and observing things through mathematical view are another essential components of mathematical creativity. Ervynck (1991) also emphasized that mathematical creativity is just advanced mathematical thinking which requires mathematical problem solvers to be familiar with the subject and making results which are framed within a deductive structure.

The reviews on the literature on the differences in mathematical creativity of professionals and students implies that students' mini-c/little-c mathematical creativity can be developed into Pro-c through formal training or informal tinkering while they learn mathematics. However, they will need to be familiar with the subjects through deep understanding of mathematical concepts in order for them to be creative. In addition, students need to be provided with opportunities to interpret familiar problems from different perspectives and suggest new solutions for given problems. Talent Development Mega Model by Subotnik, Olszewski-Kubilius, and Worrell (2011) asserts that mathematical talent and its mini-c/little-l creativity emerges earliest among several domains such as social sciences, team sports, or drawing. To facilitate its further development, it is necessary to provide opportunities early on and help the child to fall in love with mathematics. When they are in primary and secondary schools, they need opportunities to learn knowledge and skills through training, practices, and experiences. At the later stage of talent development, developing their own 'niche' will and should be guided by mentors. For full blooming of creativity, social interaction among professional experts is needed.

School Practices in Relation with Mathematical Creativity

Creative problem solving in mathematics has been emphasized in the National Council of Teachers of Mathematics (2000). However, in spite of such a significance of mathematically creative talents, mathematical education has not valued promoting students' mathematically creative thinking at schools. Review of literature informs that children need to do what mathematicians do to develop mathematical talents (Mann, 2006). However, during the last 10 years, "the worldwide emphasis on high-stakes testing has ushered in an especially virulent decade-long return to basic skills" (Lesh & Sriraman, 2005, p. 501). Teachers are under even more pressure to teach to the test rather than to work toward developing a conceptual understanding of mathematics in their students (Mann, 2006). Therefore, there is a very limited room for students to explore personal understanding; to pursue better solutions using ill-posed or open-ended problems; or to work for a prolonged period of engagement and independence.

Instead, in mathematics class, students will learn steps to follow to solve a mathematics problem directly from teachers (Mann, 2006). Rote mastery through direct instruction and practice of steps to follow to solve mathematics problems will take up most of the mathematics class time. Very little time will be spared for creative problem solving in mathematics classes. Considering various teaching approaches for enhancing student creativity explored in the last two decades (e.g., Feldhusen & Treffinger, 1980; Shallcross, 1981; Starko, 1995; Sternberg & Williams, 1996; Torrance, 1972, 1987; Treffinger, Isaksen, & Dorval, 2006), it is surprising that schools have not changed much in their efforts for nurturing creativity while teaching mathematics.

Moreover, meeting diverse needs of students in inclusive classrooms do not allow most teachers to allocate their class time for discovery learning. Therefore, limited opportunities are provided for students to work on open-ended or ill-defined tasks that require divergent thinking (Mann, 2005; Liljedahl & Sriraman, 2006) or to work on real world problems, which are ill-formed and tough (Freiman, 2006). Mathematical curriculum does not provide students with opportunities to design and answer their own problem (Mann, 2006). Therefore, these students will "struggle when they encounter unknown situations in which originality, creativity, and problem solving are necessary" (Mann, 2006, p. 248).

Finally, mathematical assessment currently used in schools generally assess mathematics computational skills, logical thinking skills, or simple application of mathematical formula to solve problems but do not identify or measure students' mathematical creativity (Cho & Hwang, 2006; Kim, Cho, & Ahn, 2003; NRC, 1989). Multiple-choice test is widely used in schools and for several purposes such as college entrance, final exams, and graduation. However, this kind of test stresses speed and accuracy instead of high-level thinking and problem solving, focuses on what answer is rather than how to find the answer (Mann, 2005). As a result, teachers, employing these tests, do not know how students think and reason and how they solve the problem. Evidently, the test values right answers over creative problem solving, although mathematics is not a fixed body of standard procedures to be mastered but rather a fluid domain (Poincare, 2012).

Currently, mathematical problems provided in school tests are often closed tasks involving numerous standard algorithms and allow only for one correct solution. This kind of problem inhibits students from exploring problem, looking for patterns, and making important mathematical generalizations that are the core of creative mathematical talents (Freiman, 2006).

The National Research Council of U.S. (1989) also concluded that the current tests do not aim to assess educational objectives and thus, teachers often teach based on test, not curriculum, or learning outcomes. Moreover, current high-stakes tests do not require higher order thinking and emphasize correct answers instead of original reasoning. Finally, tests make students, teachers and community look to mathematics as a dry and scary subject. Indeed, to develop and nurture our creative mathematical talents, current mathematical instruction, curriculum, and assessment need to be reviewed for possible revisions.

Environment has also been suggested as a critical factor for exercising creativity by various scholars. Csikszentmihalyi (1996) focused on where creativity is. He proposed that creativity exists in the interaction between an individual's thoughts and social context. Csikszentmihalyi (1996) claimed that creativity is delivered, generated, and decided through interaction among domain, field, and person. The concept of domain refers to an area that shares a set of rules and procedure (e.g., domain of mathematics). Field refers to the gatekeepers or experts who are able to decide the level of creativity of ideas, products, or performances in their domains. Person is related to an individual who creates novel and appropriate ideas. Based on these concepts, Csikszentmihalyi defined creativity as "any act, idea, or product that changes an existing domain, or that transforms an existing domain into a new one" (1996, p. 28).

Educational Implications and Strategies for Nurturing Mathematical Creativity

Although one cannot directly teach creativity, one can "teach *for* creativity" (Kaufmann & Sternberg, 2007).Based on Woods' (1990) ideas on creative teaching, Chan (2007) suggested four general features of teaching for creativity: (1) "making learning experiences relevant to students"; (2) "passing back control to the students (p. 8)"; (3) rendering students be the owner of learning and problem solving; and (4) encouraging students to be expressive and innovative. By providing students with more opportunities to control their own problem solving, students can suggest, invent, and propose ideas, make connections, and be expressive and innovative. Students will become more confident about being creative and enjoy more chances to be recognized and rewarded for their creative ideas. If students are not provided with opportunities to be creative, creativity will gradually wither (Cho, 2003, 2007). Below, more specific strategies for nurturing mathematical creativity will be reviewed.

Ensure Balanced Development of All Components Involved in Creativity

Various creativity components dynamically interact while trying to solve mathematics problems and reach to creative solutions. Each component does not function independently (Cho, 2003, 2007; Urban, 2003). Therefore, actualization of creativity can be limited by one or more of the involved components, if they are not fully developed or they do not function effectively. Even with active divergent thinking, limited understanding of basic concepts and skills will result in the ideas, solutions, and products of low quality and vice versa.

One of the mistakes that educators are prone to make in their efforts to nurture mathematical creativity of students is to focus on only divergent thinking or free association. Educators who promote teaching for creativity might be negligent of helping students to master basic mathematics knowledge and skills or to think logically. Students need to be familiar with mathematical concepts and skills in order for them to be creative (Ervynck, 1991; Liljedahl & Sriraman, 2006). Therefore, ensuring students' balanced development of each related components is necessary. Teachers need to encourage both mastery of basic math concepts and skills and production of novel ideas and solutions, especially for young children. Unbalanced focus either only on mastery of skills or divergent thinking might endanger the development of mathematical creativity.

Strategies Centered around Enhancing Motivation to be Creative

Suggested below are strategies or approaches for nurturing mathematical creativity that can be used for student who are equipped with sound and firm foundation of mathematics knowledge and skills. These strategies focus on motivation and environment which might facilitate students' higher-order thinking including divergent and convergent thinking. Csikszentmihalyi (1991) and Winner (1996) argued that the commonality among eminent innovators is not cognitive or affective but motivational, especially, persistent motivation. Several researchers (Eccles, 2006; Eccles, O'Neill & Wigfield, 2005; Graham, 2004) have presented the motivation in a dual-level view, *expectancy* and *value*, which is succinctly summarized into two questions: "Can I?" and "Do I want?"

Out of these two questions, "Can I?" is related to *expectancy*, which includes attribution, self-efficacy, and self-concept. High expectation from teachers and parents and recognition of students' strengths and efforts from teachers and parents will increase students' *expectancy*. "Do I want to do it?" is related to the sources of *value*, which includes interest in tasks (e.g., intrinsic motivation), importance or usefulness (e.g., extrinsic motivation, performance goal, mastery goal), or trade-off or cost of doing the task. Studies have found different consequences for different sources of the motivation. If any individual answers "Yes" for the two questions, then it is highly likely that they will take opportunities and commit themselves into the tasks. By employing strategies below, students are expected to respond "Yes" to the two questions, "Can I?" and "Do I want?".

Differentiation of challenge levels of mathematics problems and creating environment, which is safe for taking risks, can be employed to enhance students' perception on their expectancy. Providing more opportunities for recognition and open-ended real-life problems can contribute to enhancing students' perception on the value of mathematical creativity.

Provide Problem Choices to Differentiate Appropriately for All Individual

For effective differentiation for each individual student, it is necessary for teachers to "use of multiple resources, inquiry-based, discovery learning approaches, higher-order questioning, flexible and differentiated assessment tools, create opportunities for participation in different contests" (Freiman, 2011, p.165.). Reed (2004) also suggested three types of differentiation for mathematical creativity: extension, open-ended investigation, and self-selection of topics for in-depth study. These differentiation strategies will allow students to have choices of problems for which they want to invest their time and energy to search for creative solutions.

Some educators may think that only a few genius can be creative or should not be provided with problems which require creativity, unless they have mastered their mathematical skills. However, as the Complete 4 C Model of creativity depicts, students can and should be encouraged to exercise their creativity at any developmental level. It is because even the students with limited mathematical knowledge and skills can be creative at their own level as manifested in mini-c and little-c(Kaufmann & Beghetto, 2005).

However, it should also be noted that creativity requires profound and flexible knowledge in mathematics, long preparation and reflection (Silver, 1997; Sternberg, 2012). Students should have enough sound knowledge and skills in general and in mathematics, which will allow individuals to move forward actively interpreting problems from different perspectives, and associating remote ideas with passion. In order to encourage all students to exercise their creativity, it is necessary to differentiate the challenge level that the problems might present to the problem solver. By providing problem choices with diverse contents, students should be able to select problems, which are aligned with their interests. With choices of problems at several challenge levels, students should be able to choose problems that are aligned with their level of understanding of mathematical concepts and mastery level of mathematical skills.

Recognize Efforts and Motivational Persistence for Mathematical Creativity

Recognition of students' efforts and strengths will encourage students to be the owner who control their own learning and problem solving. Recognition and rewards are generally viewed as extrinsic motivators, which are detrimental to creativity (Amabile, 1996). But recognition of and rewards for their efforts and improvement in comparison with the learning goals or with their starting point, not with their classmates, will develop their intrinsic motivation. Teachers need to be careful not to praise students' ability and right answers only (Dweck, 2013). Rather teachers should recognize students' efforts and persistence for solving problems for which answers are not readily available; recognize students' attempt to find original and elegant solutions; recognize students' unexpected ideas, new perspectives for understanding problems, and courage to take risks of making mistakes.

Some ways to recognize these strengths and efforts is to hold competitions for solving complex/multi-step/open-ended problems and long-term competitions for solving problems whose answers are not known even to teachers. One of the examples of such a competition is International Mathematics Olympiad (Campbell, Cho, & Feng, 2011), were initiated in the former Soviet Union in 1934 as a way to find the mathematical talent that the country needed. Subsequently, these competitions have spread around the world in a similar way that sports Olympics have expanded over the years. "Virtual Mathematical Marathon," an online mathematics summer competition for mathematically promising students interested in more challenge since 2008", supported by the Canadian Mathematical Society and Canadian Natural Sciences and Engineering Research Council, (Freiman, 2011, p 167).

Extracurricular in- and after-school activities should and can be organized when mathematics competes with other traditionally attractive fields like sports and arts. These activities can include school-initiated "mathematical nights" and "Mathematics competitions" through university-school collaboration. School-initiated "Mathematical nights" can provide students in a school with opportunities to compete in chess, mathematics, and science. "Mathematics competitions" through University (Experts)- school (teachers) collaboration provides a valuable opportunity for creativity of students from wider community to be recognized. Teachers can also get benefit through interacting with mathematicians and teachers from other schools while students solve problems. These types of competitions can be organized at the local, state, national and international levels.

Psychologically Safe Environment

For students to be creative, first, students should be encouraged to express their creative ideas even though not everybody will agree with their creative ideas. Second, students should be encouraged to take sensible risks understanding that many creative ideas fail. Third, students should be encouraged to do better and see things in new ways continuously throughout life time, since creativity does not develop at once, but need to be fostered without stopping (Sternberg &Lubart, 1991).

A well-established environment for promoting creative problem solving ability should provide knowledge-based resources, stimulus, comfort, and a risk-free surrounding (Csikszentmihalyi, 1996). It is very important that parents and teachers provide a nurturing environment to develop attributes of creativity in young children. In addition, teachers and parents should provide psychologically free and safe time and space from pressure or anxiety for making mistakes and failure. Students should be able to choose to pursue self-directed research activities (Cho, 2007) without worrying about the consequences from failure. Teachers should employ instructional approaches that encourage students to generate various solutions for a problem rather than using traditional or convergent teaching model (Beghetto, 2010). Unexpected answers should be welcome; original ideas should be valued and rewarded. Moreover, teachers need to accept that model solutions offered by themselves or guidebook are not the unique ones. The best solution should be the creative one which is simple, beautiful, and useful. However, before reaching a creative solution student may have to overcome many trials and errors. Therefore, teacher should encourage students to take risks (Mann, 2006; Nadjafikhah, Yaftian, &Bakhshalizadeh, 2012). Discouraging risk taking prevents student from experiencing authentic mathematics and diminishes the development of mathematical creativity (Silver, 1997).

To stretch students' mind, Maker and Nielson (1995) suggests that classroom should be learner-centered (than teacher- or content-centered); independence (than dependence) should be expected; open to students' new ideas and innovations; acceptance than judgment exercised; complexity than simplicity should be the focus; variety of grouping options than one general grouping should be utilized; and flexible (than rigid or chaotic) class structure should be used.

In addition, teachers also need to create an environment where students can freely discuss their mathematical thought and ideas together. Plus, to be creative in mathematics, students need time for reflection and teachers should provide sufficient time for that.

Providing Students with Ill-defined and Unstructured Real World Problems

Krutetsky (1976) considered solving a problem in different ways closely connects with mathematical creativity. Open-ended problems need to be provided in the mathematical curriculum and tests because such problems stimulate students' divergent thinking, allow them approach the tasks from various ways (Nadjafikhah et al, 2012).

Challenging situations that integrate open-ended problems and mathematical investigations to evaluate and develop students' mathematical talent can nurture students' mathematical creativity. It is easier for students to feel relevant to the challenging real-life problems. A challenging situation initiates students' action of structuring a problem, and of searching for links between data and with their previous experience. Since a real challenge is possible only when the problem is non-entrenched (Kaufman & Sternberg, 2007), the challenging situation must contain a rupture with what the student has previously learned, provoking the student to reflect on the insufficiency of the past knowledge and construct new means, new mechanisms of action adapted to the new conditions, activating her full intellectual potential. It was found that working with challenging situations helps student always go further, go beyond situations, ask new questions, initiate their own investigations, and be more creative in their mathematical work (Freiman, 2006).

Students should also be provided with opportunities to do mathematics as mathematicians do in order that their mathematical creativity can be demonstrated in the mathematical classrooms. This approach should be valid considering that the only difference between the works of a school student and a mathematician in solving mathematical problems is a matter of degree (Hadamard, 1954). To reach this goal, math teachers, curriculum, and assessment should provide students with real world problems (National Mathematics Advisory Panel, 2008). Most real world problems are ill-defined (Mann, 2005). This characteristic requires students to generate multiple answers instead of a single answer (Meacham & Emont, 1989) and facilitate students to pursue different solution paths rather than only one as in closed and completed problems. Working with real world problems also enhances the students' cognitive, meta-cognitive, and argumentation skills (Shin & McGee, 2003). Indeed, doing what mathematicians do is an effective way of developing and nurturing mathematically creative talents.

Conclusion

The important goal of mathematics education is to promote highly creative professionals in science and technology. Mathematical creativity plays a major role in determining the innovative ability of the nation's work force and mathematical talents are the core of forming STEM creative professionals that create our country future (OSTP, 2006). Therefore, mathematical creativity needs to be encouraged, nurtured and efforts for nurturing mathematical creativity should be highly supported.

Mathematical creativity, at professional level, is considered the ability to generate original work that contributes greatly to our insight into mathematics and suggests important questions and ways of problem solving for other mathematicians (Liljedahl & Sriraman, 2006). In addition, creating mathematics at this level needs distinctive factors including: heuristics, intuition, proof, and social interaction (Polya, 2004). At the school level, students may reach mathematical creativity if they can offer a novel or insightful solution to a given problem as well as formulate new questions or possibilities that allow an old problem to be regarded from a new angle (Liljedahl & Sriraman, 2006).

To foster mathematical creativity at school level, students should be provided with opportunities to work with challenging open-ended or complex problems which allow and encourage students to persist to solve the problems and find new, good, and relevant solutions (NMAP, 2008). To nurture students' mathematical creativity, students should be provided with problem choices which are easily matched to their interest and their level of conceptual understanding and level of mastery of skills; their mathematical creativity should be recognized and rewarded frequently; safe environment for taking risks with enough time for reflection and persistence should be provided; and students should be provided with ill-defined challenging real life problems.

Teachers should be trained to challenge students with complex and ill-defined problems, stimulate their thinking in various ways, encourage their personal ideas, and get provoked by students' smart questions. Besides, mathematical curriculum should be reformed to provide more real world, challenging, and open-ended problems. Finally, yet importantly, mathematical assessments should not be limited to evaluating how fast students can find correct answers, but also evaluate students' mathematically creative problem solving ability. In this way, mathematically creative talents will be able to reach their fullest potential (Chan, 2007; Cho, 2003, 2007, Freiman, 2011; Mann, 2005, 2006).

Outside of the classrooms, there are still many obstacles, which might hinder the development of creativity in students in the society including requirement of conformity, strict societal hierarchy, and prevalence of standardized-test for assessment and admissions. However, the benefits of educating for creativity are worth the efforts of teachers and parents. It is hoped that students' mathematical creativity is more valued and nurtured in the coming decades with the suggested changes in classrooms and schools.

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